FractalCreator

Where art meets mathematics

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Introduction

This program is meant to calculate the Mandelbrot and Julia sets of different complex functions. It is possible to calculate Bifurcation diagrams and Julia sets of ratios of polynomials.

Theory

The Mandelbrot set is the bifurcation diagram of a complex function. This means the behavior of the function under iteration are depending on a complex parameter C that is added to the function. The critical point (where the derivative of the function is zero) is inserted for each C. Then the point C are colored differently depending on how many iterations are necessary for it to diverge outside a critical distance from the critical point. It can be shown that the dynamics of the function depends mostly on the critical point.

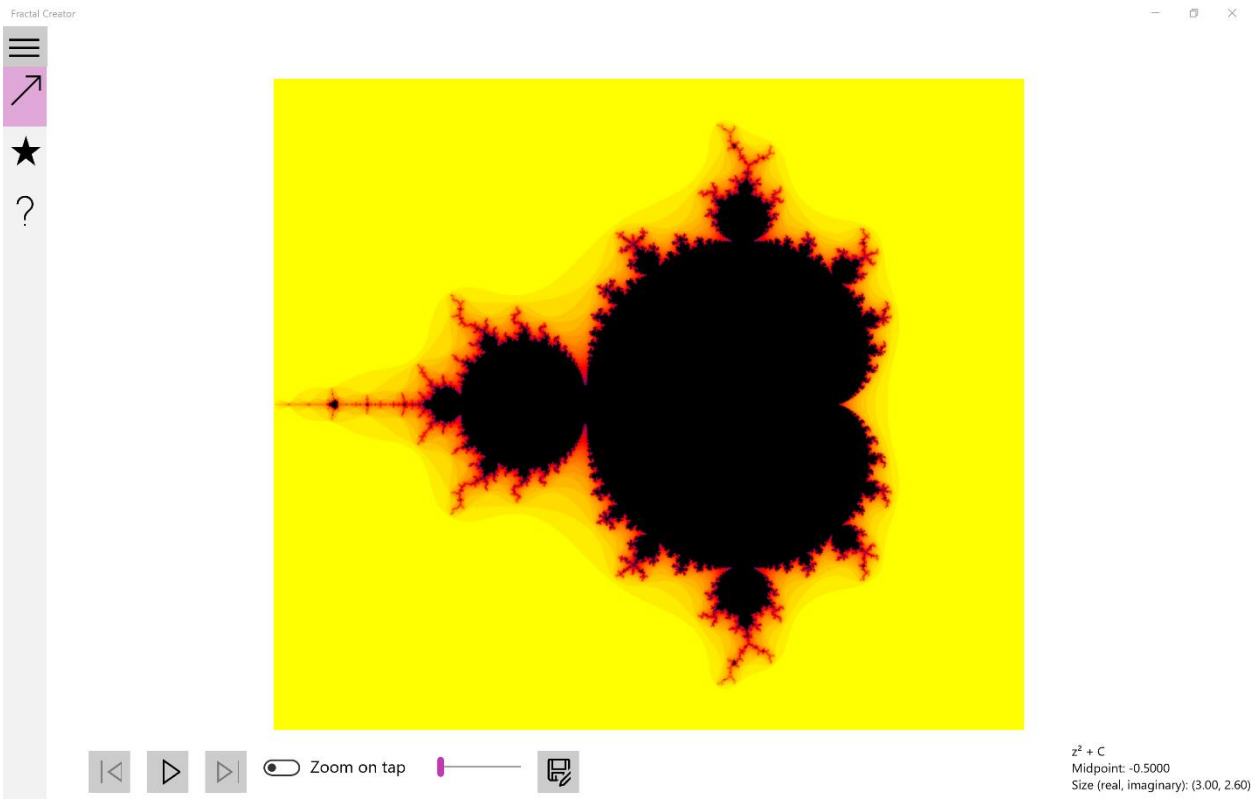
The Julia set is the set of repelling periodic points of a complex function being iterated. This set is calculated by iterating the function for different complex numbers z. Then the point corresponding to the start z is colored differently depending on the number of iterations it takes for it to reach a certain distance from the critical point. For each C in a bifurcation diagram there is a corresponding Julia set.

The application

This section describes the functionality of the application.

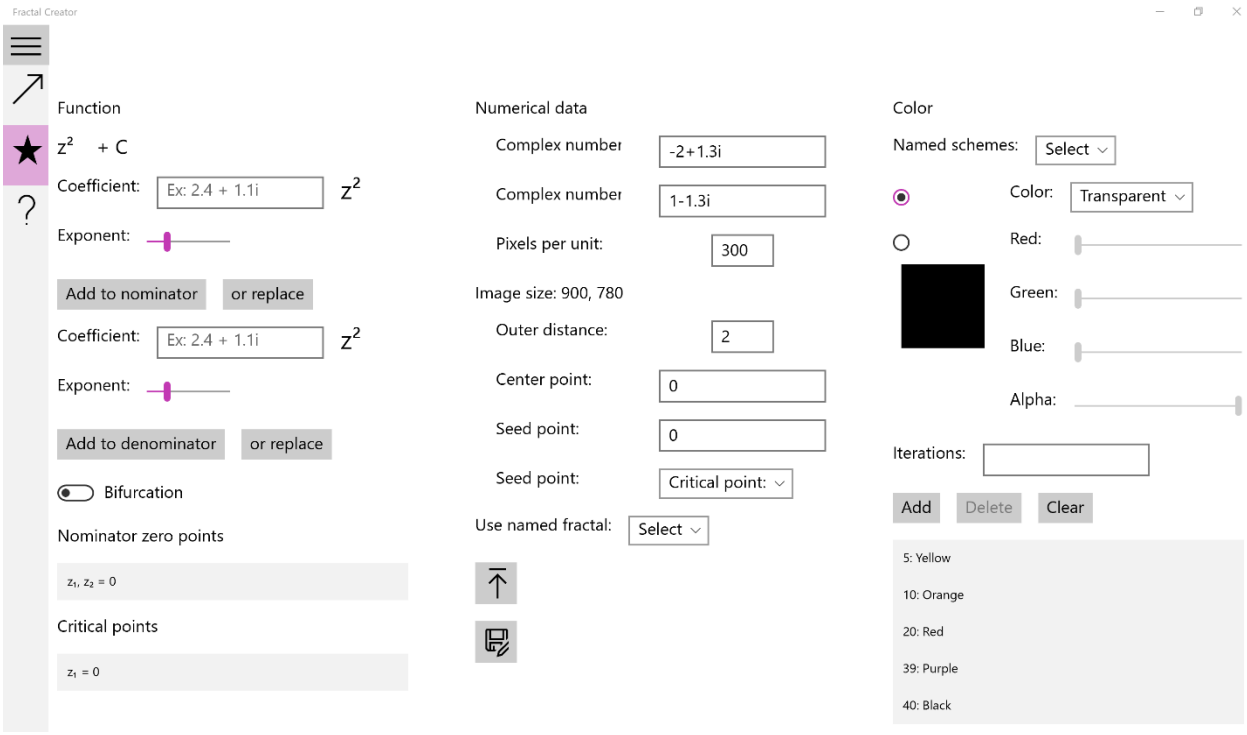
The Display page

This page shows a picture of the fractal (if calculated). There is a zooming functionality. Suppose a point is tapped somewhere on the picture. Then the area in the complex plane surrounding this point is magnified by a factor between 2 and 10 which is selected with the slider in the lower right corner. If the switch is changed the text is changed from “Zoom on tap” to “Julia set on tap” and the slider disappears. If the image is tapped the point tapped is set as C and a Julia set is calculated.



The play button means that a fractal is calculated using the values on the parameter page (reached through the star button). It will only work if correct data has been entered. The forward and backward buttons can browse through calculated fractals. When a fractal is tapped, the old fractal is kept and the new fractal is added. The old fractal can be reached through the backbutton and vice versa with the forward button. The save button opens a file save dialog where the displayed fractal can be saved as an image.

The text in the right corner contains information about the fractal. The functional form that is iterated, the midpoint of the image and the size in units in real number direction and imaginary number direction.



The Parameter page

This page sets the parameters to be used for drawing the fractal.

Column 1: Function

A complex function (sometimes called f(z)) is iterated to create a fractal. It consists of a ratio of two polynomials, f(z) = p(z)/q(z). For the Mandelbrot set: p(z) = z^2, q(z) = 1.

Nominator Polynomial

Coefficient: A complex number that is added to the coefficient before selected exponent.

Exponent: The exponent of the polynomial term to which the coefficient is added is selected with the slider.

Add button: Adds the complex number entered in Coefficient to coefficient before the term with exponent equal to the selected Exponent to the nominator polynomial.

or replace: The coefficient in the nominator is replaced instead of added.

Denominator Polynomial

Coefficient: A complex number that is added to the coefficient before selected exponent.

Exponent: The exponent of the polynomial term to which the coefficient is added is selected with the slider.

Add button: Adds the complex number entered in Coefficient to coefficient before the term with exponent equal to the selected Exponent to the denominator polynomial.

or replace: The coefficient in the nominator is replaced instead of added.

Bifurcation/Julia set switch: Determines if a bifurcation diagram or Julia set is to be calculated. (See below how choosing a Julia set changes the parameter page.)

Different data are calculated for the polynomial if possible. The zero points of the nominator and denominator expression is calculated and presented. Critical points (i.e. where the derivative of the function is zero) are calculated. They are added as potential seed points in the Seed point dropdown. Undefined points, $p'(z) = 0$ and $q'(z) = 0$ are presented separately.

Column 2: numerical data

Area in Complex Plane

Complex number 1: This is a complex number indicating the first corner of the area which will be calculated in the complex plane.

Complex number 2: This is a complex number indicating the second corner of the area which will be calculated in the complex plane.

Pixels per unit: This is the number of points calculated per unit in the complex plane this will also be the number of pixels per unit in the image.

Image size: Here the size in pixels are presented as a real number size and imaginary number size. It depends on Complex number 1, Complex number 2 and Pixels per unit. Example: We have Complex number 1=-2+1.3i, Complex number 2=1-1.3i and Pixels per unit=100. The size along the real axis will be |2-(-1)|*100=300. The size along the imaginary axis will be |-1.3-1.3|*100=260.

Outer distance: This is the absolute value of the maximum distance from the center point. When at least this distance is reached by an iteration the number of iterations are counted and compared to the colorlist to find out which color the iterated value should have.

Center point: The point from which the Outer distance is measured.

Seed Point: This is only available for bifurcation diagrams. The complex number entered here will be the number inserted at the start of each iteration chain. The dropdown with the same name adds a specific critical point to the Seed point box.

Named fractals: Contains a dropdown with certain named fractals. When selected, all parameters are set to calculate the selected fractal.

The load and save buttons can store and retrieve the parameters on this page. The parameters are stored as text files on the users computer.

If a parameter somewhere on the page is wrong a message appears below the load/save buttons.

Column 3: setting colors

Named schemes: Contains a dropdown with certain named color schemes. When selected, the color list set set corresponding to selected scheme.

The Color selector consists of two radio buttons with corresponding color selector controls. If the first one is selected, a named color can be selected in the dropdown. If the second control is selected the levels of R, G, B and alpha can be selected with sliders. The current selected color is displayed in a rectangle.

Iterations: This is the minimum number of iterations that has to be performed for a point to recieve a certain color. The color is chosen with the color selector and then the number of iterations/color is added to the colorlist with the add button.

Add: Adds a number of iterations/color pair to the color list.

Delete: deletes a selected color in the color list.

Clear: Deletes all entries in the color list.

Color list: A listbox containing the color scheme. This is a list of iterations/color pairs.

Julia sets

When the switch in the Function column indicates a Julia set the parameter page looks slightly different. The Seed point controls disappear and the following box is added in the Function column:

Julia set constant (C): Only available for Julia sets. When a Julia set is calculated the C in the bifurcation diagram is exchanged for the complex number in this box.

Philosophy

It is really amazing how very simple formulas can give rise to a wealth of complexity through iteration. I think the fractals arising from this reminds of nature. Nature often does the simplest possible but it repeats itself to some kind of fractal form. A leaf for example grows one direction. Then it splits and start anew. After a few splits the familiar leaf form appears.